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METHODS TO EVALUATE THE PERFORMANCE OF SPATIAL SIMULATION MODELS

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ABSTRACT


Quantitative methods are necessary to compare spatial patterns and evaluate the performance of spatial simulation models. We present and review several approaches to the analysis and comparison of spatial patterns. The methods are readily applicable to digital data that are in matrix (i.e., grid cell or raster) format, and include: (a) indices of particular aspects of spatial pattern, including fractal dimension, contagion, and interface; (b) spatial predictability; and (c) a variable resolution approach for measuring the degree of fit between two patterns. Because these methods measure different aspects of spatial patterns, they may be differentially suited to particular modeling and analysis objectives. In this paper, we describe the methods, apply each method to a sample data set, then evaluate the information provided and appropriate situations for its use.

INTRODUCTION

Most ecological simulation models use state variables that vary through time but are spatially aggregated (Costanza and Sklar, 1985). This approach, however, may not be adequate to address current ecological questions at large spatial scales (Risser et al., 1984; DeAngelis and Waterhouse, 1987; Meentemeyer and Box, 1987; Urban et al., 1987). For example, spatial patterning of ecosystems is important in landscape-level models (e.g., Turner, 1987, 1988; Turner et al., 1989), and interactions between spatial elements, such as the flow of energy, materials, and species among component ecosystems, must be incorporated (e.g., Kesner, 1984; Sklar et al., 1985;
Comparisons between predicted spatial patterns and actual data are thus required to evaluate spatially explicit models. Established statistical measures of goodness-of-fit can be used for relatively simple comparisons, but new methods may be required to evaluate complex spatio-temporal phenomena.

We present and review several new approaches to the analysis and comparison of spatial patterns. The methods are readily applicable to digital data that are in matrix (i.e., grid cell or raster) format, and include: (a) indices of particular aspects of spatial pattern, including complexity, contagion, interface, and anisotropy; (b) an index of predictability (Colwell, 1974) applied to spatial patterns; and (c) a variable resolution approach for cell by cell comparisons (Costanza, 1989). These methods measure different aspects of spatial patterns, and therefore they may be differentially suited to particular modeling and analysis objectives. In this paper, we describe the methods, apply each method to a sample data set, then evaluate the information and appropriate use of each technique.

METHODS

Indices of spatial pattern

A variety of indices can be used to quantify overall characteristics of spatial patterns. These indices are useful when statistical aspects of the spatial patterns must be accurately simulated, but the precise location of particular cells is less important.

Complexity of spatial patterns: Fractal dimension. Fractal analysis (Mandelbrot, 1977, 1983) was introduced as a method to study spatial patterns that are similar when observed at many scales (i.e., ‘self-similar’). Boundaries or shapes can be quantified using fractals, and the fractal dimension can then be used as a measure of the complexity of spatial patterns. This application has been useful in studies of landscape patterns, the spatial patterns resulting from physical, biological and human forces over a geographic area. Fractals have been used to compare simulated and actual landscapes (Gardner et al., 1987; Turner, 1987), to compare the geometry of different landscapes (Krummel et al., 1987; Milne, 1988; O’Neill et al., 1988; Turner and Ruscher, 1988), and to judge the relative benefits to be gained by changing scales in a model or data set (Burrough, 1986). A perimeter to area relationship can be used to calculate the fractal dimension of patch perimeters using grid cell data (Burrough, 1986; Gardner et al., 1987). Using all patches of a single cover type (or all cover types) in a landscape scene, a regression is calculated between log (perimeter/4), the length scale used in measuring the regression line. The dimension of a random pattern from the regression line. The dimension of a random pattern is 2.0, and higher values representing nested patterns are calculated by dividing by the total number of patches of type i being aggregated. The probability that type i is distributed over a grid cell is the number of types, j. (1988) used these probability calculations for the patterns in Georgia.

Adjacency and contagion: Indices of spatial dependence are calculated by dividing the number of land cover type i being aggregated. The probability that type i is distributed over a grid cell is the number of types, j. (1988) used these probability calculations for the patterns in Georgia.

Proportions of adjacency across grid cells are distilled to a single index of spatial dependence:

\[ D_2 = 2s \log s + \sum_{i=1}^{m} \sum_{j=1}^{n} q_{ij} \]

The contagion index, \( D_2 \), is an aggregated or clumped. A value of 1 from the equiprobable model, and large, contiguous areas of land. The value of \( D_2 \), the landscape is distributed among grid cells of the same type. The difference in the numbers of land cover types of adjacency can indicate:

Interface: Edges between different spatial components may influence materials across boundaries (Gardner et al., 1987), and the importance of this should not be underestimated. Thus, it may be important...
predicted spatial patterns and by explicit models. Established be used for relatively required to evaluate complex placed to the analysis and saly applicable to digital format, and include: (a) luding complexity, contanictability (Colwell, 1974) ution approach for cell by measure different aspects rationally suited to is paper, we describe the a set, then evaluate the

The overall characteristics of statistical aspects of the the precise location of

Dimesion. Fractal analysis method to study spatial scales (i.e., 'self-similar'). fractals, and the fractal the complexity of spatial processes of landscape patterns, logical and human forces (to compare simulated and , 1987), to compare the al., 1987; Milne, 1988; and to judge the relative el or data set (Burrough, ed to calculate the fractal Burrough, 1986; Gardner (or all cover types) in a corner log (perimeter/4), the

length scale used in measuring the perimeter and log (size) of each patch. The fractal dimension of the patch perimeters then equals twice the slope of the regression line. The dimension can range between 1.00 and 2.00, with higher values representing more convoluted boundaries; the expected fractal dimension of a random pattern is 1.5. Calculating the fractal dimension with least-squares is quite reliable, with the $r^2$ values generally exceeding 0.95.

Adjacency and contagion: Nearest neighbor probabilities. Nearest neighbor probabilities, $q_{i,j}$ – sometimes referred to as Markovian spatial transition probabilities (Lin and Harbaugh, 1984) – represent the probability of cells of land use type $i$ being adjacent to cells of land use type $j$. The $q_{i,j}$ values are calculated by dividing the number of cells of type $i$ that are adjacent to type $j$ by the total number of cells of type $i$. A landscape with very large patches of type $i$ will have a relatively high $q_{i,j}$; however, if the same area of type $i$ is distributed over many small patches, the $q_{i,j}$ will be low. Turner (1988) used these probabilities to compare simulated and actual landscape patterns in Georgia.

Probabilities of adjacency are relative easy to calculate with a single pass through a matrix of land cover types. Adjacency information can also be distilled to a single index of the overall contagion (O’Neill et al., 1988) on an $m \times n$ landscape containing $s$ cover types using the formula:

$$D_2 = 2s \log s + \sum_{i=1}^{m} \sum_{j=1}^{n} q_{i,j} \log q_{i,j}$$  

The contagion index, $D_2$, measures the extent to which land uses are aggregated or clumped. At high values of $D_2$, the summation term deviates from the equiprobable maximum in which all adjacency probabilities are equal, and large, contiguous patches are found in the matrix. At low values of $D_2$, the landscape is dissected into many, small patches.

It may also be of interest to compare nearest neighbor probabilities that are calculated both vertically and horizontally such that anisotropism, or directionality, in the spatial pattern can be measured. Directional probabilities are determined by dividing the number of cells of type $i$ that are horizontally or vertically adjacent to cells of type $j$ by the total number of cells of type $i$. The difference between the horizontal and vertical probabilities of adjacency can indicate directional alignment of spatial components.

Interface: Edges between components. The amount of edge between different spatial components may be important for the movement of organisms or materials across boundaries (e.g., Wiens et al., 1985; Turner and Bratton, 1987), and the importance of edge habitat for various species is well known. Thus, it may be important to monitor edges when predicting spatial patterns.
and when integrating pattern with function. Edges can be simply calculated by adding both vertical and horizontal edges of cells between land uses and multiplying by the length of the side of a cell.

**Spatial predictability**

Information theoretic concepts were applied to estimating the degree of predictability of periodic phenomena by Colwell (1974). Predictability in this context refers to the reduction in uncertainty about one variable that can be gained from knowledge of another. For example, if the seasonal rainfall pattern in an area is predictable (e.g., there is always a severe dry summer), then knowing the time of year provides information about rainfall (if it’s summer, it’s probably dry). If there is no relationship between rainfall and season, the rainfall is relatively unpredictable from knowledge of the time of year. Application of these techniques to spatial data measures the reduction in uncertainty about the state of a particular pixel obtained from other knowledge about the pattern. Regularities in spatial data are identified and ranked on a scale from 0 (unpredictable) to 1 (predictable). The result may be interpreted as the degree of departure of the scene from a random

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
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<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
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<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
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<td>11</td>
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<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
</tr>
</tbody>
</table>

**TABLE 1**

Numerical data for the 20×20 base matrix (Fig. 1A)

Integers represent different data categories, such as land cover.
can be simply calculated between land uses and elements (unpredictable) pattern. We develop two measures: (a) spatial adjacency predictability, based on the state of adjacent pixels (similar to the contagion index, but including higher level adjacency information, i.e., the state of adjacent pairs, triplets, quartuples etc.); and (b) spatial address predictability, based on the row or column address of the pixel.

Fig. 1. Test data for demonstrating the methods. All matrices are $20 \times 20$ arrays with three categories (e.g., land cover, vegetation, etc.) each. Matrices A through G have the same proportions of each category (category 1 = 0.45; category 2 = 0.35; category 3 = 0.20). Matrix H has an approximately equiprobable distribution of the three categories.
TABLE 2
First-order contingency table used to calculate spatial predictability for the 20×20 matrix in Table 1

<table>
<thead>
<tr>
<th>Category</th>
<th>Category</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>Totals $Y_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>187</td>
<td>104</td>
<td>55</td>
<td>346</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>102</td>
<td>161</td>
<td>8</td>
<td>271</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>52</td>
<td>2</td>
<td>89</td>
<td>143</td>
</tr>
<tr>
<td>Totals ($X_j$)</td>
<td></td>
<td>341</td>
<td>267</td>
<td>152</td>
<td>760</td>
</tr>
</tbody>
</table>

Entries ($N_{ij}$) are the frequency that the column category is followed by the row category, either horizontally or vertically (e.g., category 1 followed by category 2, 102 times; category 2 followed by category 1, 104 times). First-order predictability ($P_1$) = 0.222.

Note that for an $m \times m$ matrix, the total is equal to $2 \cdot [m \times (m-1)]$ which is 2 (20 - 19) in this example.

To estimate spatial adjacency predictability, a contingency matrix is developed with rows corresponding to the states of the pixels, and columns corresponding to groups of $n$ pixels. For example, using numerical data of the scene in Fig. 1A (Table 1), first-order ($n = 1$; Table 2) and second-order ($n = 2$; Table 3) contingency matrices were developed. The rows represent the three cover types, and the matrix entries are the number of times each row category occurred adjacent to the column category (or category pair [as in Table 2], triplet, etc. in higher-order analyses) represented by each column. The second-order contingency matrix (Table 3) incorporates greater detail than the first, tabulating the number of times that an ordered pair of categories is adjacent to another category (e.g., how frequently are two forest pixels followed by a grassland pixel). Higher-order matrices continue to incorporate more information, representing the frequency that an ordered group of $n$ pixels is adjacent to a particular category.

TABLE 3
Second-order contingency table for the 20×20 matrix in Table 1

<table>
<thead>
<tr>
<th>Category</th>
<th>Category pairs</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>21</th>
<th>22</th>
<th>23</th>
<th>31</th>
<th>32</th>
<th>33</th>
<th>Totals $Y_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>86</td>
<td>29</td>
<td>12</td>
<td>58</td>
<td>66</td>
<td>0</td>
<td>31</td>
<td>3</td>
<td>36</td>
<td>321</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>57</td>
<td>68</td>
<td>3</td>
<td>30</td>
<td>85</td>
<td>0</td>
<td>13</td>
<td>5</td>
<td>5</td>
<td>266</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>33</td>
<td>0</td>
<td>34</td>
<td>10</td>
<td>2</td>
<td>2</td>
<td>9</td>
<td>0</td>
<td>43</td>
<td>133</td>
</tr>
<tr>
<td>Totals ($X_j$)</td>
<td></td>
<td>176</td>
<td>97</td>
<td>49</td>
<td>98</td>
<td>153</td>
<td>2</td>
<td>53</td>
<td>8</td>
<td>84</td>
<td>720</td>
</tr>
</tbody>
</table>

Entries ($N_{ij}$) are the frequency that the column category (ordered pairs) is followed by the row category (either horizontally or vertically). Second-order predictability ($P_2$) = 0.228.

Note that for an $m \times m$ matrix, the total is equal to $2 \cdot [m \times (m-2)]$ which is 2 (20 - 18) in this example.

To estimate predictability, use matrix (address predictability by cross) entries corresponding to the contingency matrix representing the use matrix (e.g., rows $1$ the occurrences of each state be sensitive to banded values.

Following Colwell (1977), if the use matrix of size $s$ by $t$, $X_{ij}$, the grand total, or:

$$X_j = \sum_{i=1}^{s} N_{ij}$$
$$Y_j = \sum_{j=1}^{t} N_{ij}$$
$$Z = \sum_{i=1}^{s} \sum_{j=1}^{t} N_{ij}$$

The uncertainty with respect to $X_j$:

$$H(X) = -\sum_{j=1}^{t} \frac{X_j}{Z} \log \frac{X_j}{Z}$$

and the uncertainty with respect to $Y_j$:

$$H(Y) = -\sum_{i=1}^{s} \frac{Y_i}{Z} \log \frac{Y_i}{Z}$$

and the uncertainty with respect to $XY$:

$$H(XY) = -\sum_{i} \sum_{j} \frac{N_{ij}}{Z} \log \frac{N_{ij}}{Z}$$

Note that the first-order nearest neighbor probability unconditional uncertainty of:

$$H_s(Y) = H(XY) - H(X)$$

Finally, a measure of predictability as:

$$P = 1 - \frac{H_s(Y)}{\log s} = 1 - \frac{H(XY) - H(X)}{\log s}$$

Predictability will be smaller if $s$ is large.
To estimate predictability based on the location of the pixel in the land use matrix (address predictability), the contingency matrix is developed with rows corresponding to the states of the pixels. The columns in the contingency matrix represent the row or column location of a pixel in the land use matrix (e.g., rows 1 through 20). Matrix entries represent the number of occurrences of each state in each row or column. Address predictability may be sensitive to banded patterns in the data.

Following Colwell (1974) we define \( N_{ij} \) as the elements in the contingency matrix of size \( s \) by \( t \), \( X_j \) as the column totals, \( Y_i \) as the row totals, and \( Z \) as the grand total, or:

\[
X_j = \sum_{i=1}^{s} N_{ij}
\]

\[
Y_i = \sum_{j=1}^{t} N_{ij}
\]

\[
Z = \sum_{i=1}^{s} \sum_{j=1}^{t} N_{ij}
\]

The uncertainty with respect to \( X \) is:

\[
H(X) = -\sum_{j=1}^{t} \frac{X_j}{Z} \log \frac{X_j}{Z}
\]

and the uncertainty with respect to \( Y \) is:

\[
H(Y) = -\sum_{i=1}^{s} \frac{Y_i}{Z} \log \frac{Y_i}{Z}
\]

and the uncertainty with respect to the interaction of \( X \) and \( Y \) is:

\[
H(XY) = -\sum_{i}^{s} \sum_{j}^{t} \frac{N_{ij}}{Z} \log \frac{N_{ij}}{Z}
\]

Note that the first-order contingency matrix is related to the matrix of nearest neighbor probabilities \( Q \) by the relationship \( N_{ij}/X_j = q_{ij} \). The conditional uncertainty of \( X \) given \( Y \) is:

\[
H_x(Y) = H(XY) - H(X)
\]

Finally, a measure of predictability \( P \) with the range \( (0, 1) \) can be defined as:

\[
P = 1 - \frac{H_x(Y)}{\log s} = 1 - \frac{H(XY) - H(X)}{\log s}
\]

Predictability will be minimal when all the elements in the contingency
matrix \((N_j)\) are equiprobable (i.e., when all entries are the same) and will be maximized when only one entry in each column is non-zero. Most spatial data will fall between these extremes. For example, the contingency matrix in Table 2 has a predictability of 0.222 indicating that if the state of a given pixel is known, the uncertainty about the state of the following pixel (either horizontally or vertically) is reduced by 22.2%. Highly clumped patterns will have a high adjacency predictability. First-order spatial predictability \((P_1)\) and the contagion index \((D_2)\) yield similar, but not identical, information. However, \(P\) is scaled on the range \((0, 1)\) whereas \(D_2\) is not.

**Multiple resolution goodness-of-fit**

A cell by cell comparison between a model’s predicted spatial patterns and the actual patterns is necessary if the location of different habitats relative to their actual locations is important. However, a comparison done only at a fine resolution may not adequately evaluate the prediction. A standard fit procedure can be applied at a number of spatial resolutions, and the change in fit with resolution of the sampling window may be a better tool for interpreting the patterns predicted by a model (Costanza, 1989). We use an algorithm for this purpose that gradually decreases the resolution of comparison by increasing the size of the sampling window used to calculate the fit. For a sampling window size of one cell (the cell by cell comparison), the fit is the proportion of cells that are correctly matched, regardless of their spatial arrangement. For example, if a particular \(2 \times 2\) window had two cells of forest and two of marsh in both scenes, the fit would be 100% regardless of how the cells were arranged within the windows. If one sampling window had one forest and three marsh, while the other had two of each category, the fit would be 75% (three out of four were correct). The sampling window is moved through the scene one cell at a time until the entire image is covered. The average fit over all sampling windows of a particular size is then calculated, representing the overall fit at that resolution.

The formula for the fit at a particular sampling window size \((F_w)\) is:

\[
F_w = \frac{\sum_{s=1}^{t_w} \left( \frac{\sum_{i=1}^{p} |a_{1i} - a_{2i}|}{2w^2} \right) s}{t_w}
\]

where \(F_w\) is the fit for sampling window size \(w\), \(w\) the dimension of one side of the (square) sampling window, \(a_{ji}\) the number of cells of category \(j\) in scene \(k\) in the sampling window, \(p\) the number of different categories (e.g., habitat types) in the sampled scene \(s\) by \(w\) which slides until the window size would reach \(F_w = 1.0\). If the curve would increase \(F_w = 1.0\), indicating that the randomly generated scene fit would start at 1/s and the window encompassed the entire image.

An overall index of fit obtained at different window sizes is obtained using the following formula, different from the above:

\[
F_t = \frac{\sum_{w=1}^{n} F_w e^{-k(w-1)}}{\sum_{w=1}^{n} e^{-k(w-1)}}
\]

where \(F_t\) is weighted average of sampling windows of limited dimension of a sampling grid, each of the same weight. When \(k\) is high, the value of \(k\) can be added to the sampling quality of the data.

**DATA**

Sample \(20 \times 20\) matrices were used as described above. Matrices were altered to represent any kind of misregistration \((p_1 = 0.45; p_2 = 0.45; p_3 = 0.35); p_3\), cover types vary. Matrix \(A\) was altered (Fig. 1) by: misregistration (A); by flipping the matrix (B); by flipping the matrix (C); by creating a high dimensional pattern that maintains the overall size of the pattern (Fig. 1G). Matrix \(H\) is a class of妄等、等量な比率 \((p_1 = p_2 = p_3)\).
spatial predictability \( P_1 \) is identical, information \( P_2 \) is not.

The predicted spatial patterns of different habitats, however, a comparison done to evaluate the prediction. A lower spatial resolutions, and if the window may be a better model (Costanza, 1989). We increase the resolution of window used to calculate (cell by cell comparison), matched, regardless of particular \( 2 \times 2 \) window had \( F_w = 1.0 \), the fit would be 100% in the windows. If one while the other had two four were correct). The cell at a time until the sampling windows of a overall fit at that resolution window size \( F_w \) is:

\[
F_w = \frac{\sum_{w=1}^{n} F_w e^{-k(w-1)}}{\sum_{w=1}^{n} e^{-k(w-1)}}
\]

where \( F_i \) is weighted average of the fits over all window sizes, \( F_w \) the fit for sampling windows of linear dimension \( w \), \( k \) a constant, and \( w \) linear dimension of a sampling window. When \( k = 0 \), all window sizes are given the same weight. When \( k = 1 \), only the smaller window sizes are important. The value of \( k \) can be adjusted depending on the model objectives and the quality of the data.

**DATA**

Sample \( 20 \times 20 \) matrices (Fig. 1) were used to compare the methods described above. Matrices A through G contain three cover types (considered to represent any kind of categorical data) in the same proportions \( p_1 = 0.45; p_2 = 0.35; p_3 = 0.20 \). However, the spatial arrangements of the cover types vary. Matrix A (Fig. 1A) is the base case, which is then altered (Fig. 1) by: misregistration (B); by creating anisotropy in one cover type (C); by flipping the matrix vertically (D); by creating a fragmented pattern (E); and by creating a highly clumped pattern (F). Matrix G is a random pattern that maintains the same category proportions as the other matrices (Fig. 1G). Matrix H is a checkerboard (Fig. 1H) with the three categories in equal proportions \( p_1 = p_2 = p_3 = 0.33 \).
RESULTS

Indices of spatial patterns

The fragmented and random matrices (Fig. 1E and G) exhibit the most complex spatial patterns as measured by the fractal dimension (Table 4). The anisotropic matrix (Fig. 1C) has a relatively simple overall pattern, reflected by a fractal dimension of 1.469, although the clumped pattern should give a lower result if there was a sufficient number of patches to use for the calculations. The checkerboard matrix (Fig. 1H) has the most simple patch shapes (all squares), whose perimeters have a fractal dimension of 1.0, by definition. The fractal dimensions in matrices A, B and D show little difference, reflecting the same complexity in cover type 2: a fractal dimension of 1.199.

The probabilities of adjacency differences in pattern than the vertical q_{1i}’s in matrix B. When A is misregistered by 90°, all patch weights change slightly while the vertical adjacencies are sensitive to slight changes of the spatial component is also misregistered and vertical adjacencies. For example, using the 3 adjacent to 3 in matrix C (Fig. 1C) is increased from 0.615 (0.813) and q_{33} horizontally adjacent is increased from 0.128 to 0.524. The patterns are very fragmented. The contagion index is low or high, respectively, in horizontal and vertical directions. This pattern shows a difference in horizontal and vertical directions.

The contagion index of the clumped (D_2 = 4.891) matrix is a clear indicator of broad patterns in the data. The vertical inversion (Fig. 1F) checkerboard pattern also has a more clumped pattern. Although contagion is anisotropic, anisotropy can be seen in the horizontal contagion values. However, the cause of directionality cannot be determined using these measures.

The amount of edge in individual matrices is shown in Table 4. The edge values of the matrices are similar although the location of the edge values is related to the contagion index. Matrices A and D have many edges (matrices B and C have few edges (matrix F).

Spatial predictability

Matrices A through D all have a P_1 of approximately 5); all have a P_1 of approximately 5. These spatial patterns are statistically significant, with the fragmented and random matrices showing the highest predictability.

The predictabilities measure...
E and G) exhibit the most simple dimension (Table 4). The number of patches to use (Fig. 1H) has the most simple fractal dimension of 1.0, and A, B and D show little difference, reflecting the similarity in their spatial patterns. The decreased complexity in cover type 3 in the anisotropic matrix is measured by a low fractal dimension of 1.199.

The probabilities of adjacency (Table 4) appear more sensitive to fine differences in pattern than the fractal dimension. All horizontal $q_{i,j}$'s exceed the vertical $q_{i,j}$'s in matrix A, indicating a slight horizontal orientation. When A is misregistered by two columns, as in matrix B, the horizontal $q_{i,j}$'s change slightly while the vertical $q_{i,j}$'s remain the same. Thus, the probabilities are sensitive to slight changes in spatial adjacencies. The anisotropy of a spatial component is also readily apparent in differences between horizontal and vertical adjacencies. For example, the vertical orientation of cover type 3 in matrix C (Fig. 1C) is reflected in the difference between $q_{3,3}$ vertical (0.813) and $q_{3,3}$ horizontal (0.387) (Table 4). In contrast, when spatial patterns are very fragmented (matrix E) or clumped (matrix F), the $q_{i,j}$'s are low or high, respectively, but show little difference between the horizontal and vertical directions. The isotropic pattern (matrix H) exhibits no difference in horizontal and vertical adjacencies.

The contagion index differentiates the fragmented ($D_2 = 3.612$) and clumped ($D_2 = 4.891$) matrices (Table 4), suggesting that $D_2$ may be an adequate indicator of broad-scale pattern. However, $D_2$ is not sensitive to the vertical inversion (Fig. 1D) or to misregistration (Fig. 1B). The checkerboard pattern also has a relatively high contagion value (4.516) indicating a clumped pattern. Although contagion does not necessarily reflect directionality, anisotropy can be identified in differences between vertical and horizontal contagion values (4.389 and 3.877, respectively, in matrix C). However, the cause of directionality (i.e., which components are anisotropic) cannot be determined using $D_2$ alone.

The amount of edge in the matrices is not sensitive to the precise spatial patterns (Table 4). The edges present in matrices A, B, C and D are quite similar although the locations of the patches vary. Edges values are inversely related to the contagion in the matrix, such that patterns with low contagion have many edges (matrices E and G) and patterns with high contagion have few edges (matrix F).

**Spatial predictability**

Matrices A through D are similar in their adjacency predictability (Table 5); all have a $P_1$ of approximately 0.2. This is not surprising because their spatial patterns are statistically similar (Table 4). The adjacency predictability clearly differentiates matrices A–D from the clumped matrix ($P_1 \approx 0.47$) and the fragmented and random matrices ($P_1 \approx 0.07$ and 0.04, respectively). The predictabilities measured in the random and fragmented matrices ap-
### TABLE 5

Summary of predictability measures applied to the test data

<table>
<thead>
<tr>
<th>Matrix</th>
<th>Misregistered</th>
<th>Anisotropic</th>
<th>Flipped vertically</th>
<th>Fragmented</th>
<th>Clumped</th>
<th>Random</th>
<th>Checkerboard</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.2219</td>
<td>0.2042</td>
<td>0.2062</td>
<td>0.2178</td>
<td>0.0751</td>
<td>0.4674</td>
<td>0.0376</td>
</tr>
<tr>
<td>B</td>
<td>0.2283</td>
<td>0.2133</td>
<td>0.2103</td>
<td>0.2222</td>
<td>0.0826</td>
<td>0.4799</td>
<td>0.0447</td>
</tr>
<tr>
<td>C</td>
<td>0.2617</td>
<td>0.2692</td>
<td>0.2601</td>
<td>0.2589</td>
<td>0.1102</td>
<td>0.5266</td>
<td>0.0739</td>
</tr>
<tr>
<td>D</td>
<td>0.2999</td>
<td>0.3024</td>
<td>0.3093</td>
<td>0.3030</td>
<td>0.1923</td>
<td>0.5548</td>
<td>0.1586</td>
</tr>
<tr>
<td>Spatial adjacency predictability</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_1$</td>
<td>0.120</td>
<td>0.120</td>
<td>0.222</td>
<td>0.120</td>
<td>0.067</td>
<td>0.221</td>
<td>0.066</td>
</tr>
<tr>
<td>$P_2$</td>
<td>0.129</td>
<td>0.129</td>
<td>0.097</td>
<td>0.129</td>
<td>0.081</td>
<td>0.144</td>
<td>0.083</td>
</tr>
<tr>
<td>$P_3$</td>
<td>0.124</td>
<td>0.124</td>
<td>0.159</td>
<td>0.124</td>
<td>0.074</td>
<td>0.182</td>
<td>0.074</td>
</tr>
<tr>
<td>$P_{avg}$</td>
<td>0.130</td>
<td>0.130</td>
<td>0.163</td>
<td>0.130</td>
<td>0.076</td>
<td>0.213</td>
<td>0.076</td>
</tr>
</tbody>
</table>

$P_1$ through $P_4$ refer to the predictability of the state of a pixel given the states of groups of one, two, three, and four adjacent pixels. $P_x$ and $P_y$ refer to the predictability of a pixel given only its row ($x$) or column ($y$). $P_{avg}$ is the average of $P_x$ and $P_y$.

- **A**: Checkerboard 1
  - $P_1 = 1.00$
  - $P_2 = 1.00$
  - $P_3 = 1.00$
  - $P_4 = 1.00$

- **B**: Checkerboard 2
  - $P_1 = 0.37$
  - $P_2 = 1.00$
  - $P_3 = 1.00$
  - $P_4 = 1.00$

Fig. 2. Repeating checkerboard patterns at different base scales. Both patterns are predictable, but this is not apparent in B until the second order predictability analysis is done because the pattern is composed of 2×2 blocks of pixels.

and $P_y = 0.097$), reflecting the nature of matrix C. The checkerboard matrix X and the row or column address predictability on column are statistically identical.

There are no large breaks through F, and higher order analysis was not applicable. Additional information above and below this level analysis does elicit significant predictability.

In the checkerboard matrices $P_y$ and $P_{avg}$ indicating complete predictability of the row and column predictability of a pixel can identify the spatial scale of the checkerboard pattern. The pattern repeats every two pixels in both row (x) and column (y) $P_{avg}$ for the pattern in Fig.2. This first-order analysis. The predictability of the pixel in the first-order analysis ($P_1 = 0.37$) below this level, but the second order analysis is significant. Therefore, the most appropriate model to use is the checkerboard to identify patterns that repeat in both directions.

**Multiple resolution fitting**

Results of the multiple resolution fitting indicate that the base map ($k = 0.1, F_r = 0.914$). The $F_r$ for the map ($k = 0.1, F_r = 0.914$). The $F_r$ for the map ($k = 0.1, F_r = 0.914$).

**TABLE 6**

Goodness-of-fit between the original resolution fit procedure and different models, which is the predictability of a given map

**Comparison**

|----------------------|---------------------|---------------------------|-------------------|----------------|-------------------|--------------------------|---------------------------|

- A × B: Misregistered
- A × C: Anisotropic
- A × D: Flipped vertically
- A × E: Fragmented
- A × F: Clumped
- A × G: Random
- G × G: Random × Random
- E × G: Fragmented × Random
and $P_Y = 0.097$), reflecting the vertical bands that were introduced into matrix $C$. The checkerboard matrix is least predictable from knowledge of the row or column address of a cell ($P_x = P_y = 0.009$), because each row and column are statistically identical.

There are no large breaks or periodicities in the patterns in matrices $A$ through $F$, and higher order adjacency predictabilities ($P_1 - P_3$) yield little additional information about these patterns (Table 5). However, the multilevel analysis does elicit significant additional information in certain cases. In the checkerboard matrix, second-order spatial predictability is 1.00, indicating complete predictability at that scale. Thus, spatial predictability can identify the spatial scales of periodicities in a pattern. For example, the scale of the checkerboard pattern is one pixel in Fig. 2A, whereas the pattern repeats every two pixels in Fig. 2B. Spatial predictability ($P_1$ through $P_4$) is 1.0 for the pattern in Fig. 2A, as the periodicity is apparent even in the first-order analysis. The pattern in Fig. 2B has low predictability with a first-order analysis ($P_1 = 0.370$) because the periodicity is not apparent at this level, but the second order analysis indicates complete predictability. Therefore, the most appropriate use of the higher-order predictabilities may be to identify patterns that show periodicity at different spatial scales.

**Multiple resolution fitting**

Results of the multiple resolution fitting analyses (Table 6, Fig. 3) indicate that the base matrix is most similar to the anisotropic matrix (at $k = 0.1$, $F_i = 0.914$). The positions of pixels of category 2 were retained.

<table>
<thead>
<tr>
<th>Comparison</th>
<th>Weighted average of goodness-of-fit ($F_i$)</th>
<th>Cross predictability $P_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$k = 0.0$</td>
<td>$k = 0.01$</td>
</tr>
<tr>
<td>A × B: Misregistered</td>
<td>0.821</td>
<td>0.815</td>
</tr>
<tr>
<td>A × C: Anisotropic</td>
<td>0.930</td>
<td>0.928</td>
</tr>
<tr>
<td>A × D: Flipped vertically</td>
<td>0.793</td>
<td>0.787</td>
</tr>
<tr>
<td>A × E: Fragmented</td>
<td>0.856</td>
<td>0.852</td>
</tr>
<tr>
<td>A × F: Clumped</td>
<td>0.709</td>
<td>0.702</td>
</tr>
<tr>
<td>A × G: Random</td>
<td>0.768</td>
<td>0.761</td>
</tr>
<tr>
<td>G × G: Random × Random</td>
<td>0.801</td>
<td>0.800</td>
</tr>
<tr>
<td>E × G: Fragmented × Random</td>
<td>0.829</td>
<td>0.823</td>
</tr>
</tbody>
</table>
Fig. 3. Fit versus window size for various pairs of the matrices in Fig. 1 using the multiple resolution fitting method. For example, AxB is the comparison of the base matrix (A) with the misregistered matrix (B).

when anisotropy was introduced, and only a small fraction of the other two categories was rearranged. Thus, a relatively high cell by cell agreement is observed between the anisotropic and base matrices.

The fragmented pattern exhibits the next best fit (at $k = 0.1$, $F_i = 0.821$) with the base matrix. This seems counter-intuitive, because the fragmented and random patterns are similar (Tables 4 and 5). However, the fragmented pattern was created from the base matrix by arbitrarily rearranging some of the pixels in Fig. 1A and not by randomly creating a new pattern. Therefore, more pixels in the fragmented pattern correspond to pixels in the base matrix ($\approx 68\%$) than expected at random ($\approx 33\%$), even though the fragmented and random patterns appear similar.

The base matrix does not fit well with the clumped or random patterns, as might be expected. However, the misregistered and vertically flipped matrices also do not fit well with the base matrix, although their patterns are identical except for slight misregistration. This result indicates a limitation of the multiple resolution fitting procedure and suggests the need to handle misregistration problems separately (Costanza, 1989). If registration between patterns is questionable, a cross-correlation analysis could be used to determine the point of maximum registration. After the maps have been registered, the multiple resolution fitting procedure should produce more reasonable results.

Cross predictability ($P_c$) can be calculated for pairs of maps to determine the predictability of the state of pixels on one map given their corresponding state on another map. This is another goodness-of-fit test and the results for our example maps are included in Table 6. This index of fit correlates well with the multiple resolution test ($r^2 = 0.918$ for $k = 1.0$) but yields a different perspective and range of values. Values for this test range from approximately 0.04 for core map compared with the anisotropic category of a pixel or about its state on the anisotropic or

DISCUSSION

What information is provided relative cost to calculate each. The indices of spatial patterns, contagion, edges, spread, whether certain aspects of whether the cell by cell pattern may be adequate when one

<table>
<thead>
<tr>
<th>Measure</th>
<th>Character</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fractal dimension</td>
<td>complexity</td>
</tr>
<tr>
<td></td>
<td>spatial pattern</td>
</tr>
<tr>
<td>Nearest neighbor</td>
<td>adjacency</td>
</tr>
<tr>
<td>probabilities</td>
<td>anisotropy</td>
</tr>
<tr>
<td>Contagion index</td>
<td>fragmentation or clumping</td>
</tr>
<tr>
<td>Edges</td>
<td>edge between</td>
</tr>
<tr>
<td>Spatial predictability</td>
<td>departure from random</td>
</tr>
<tr>
<td></td>
<td>periodicity, different</td>
</tr>
<tr>
<td>Multiple resolution</td>
<td>correspondence</td>
</tr>
<tr>
<td>fitting</td>
<td>actual locations</td>
</tr>
</tbody>
</table>
approximately 0.04 for comparisons with random maps to 0.66 for the base map compared with the anisotropic case. This indicates that knowledge of the category of a pixel on the base map removes 66% of the uncertainty about its state on the anisotropic map, but this knowledge removes only 4.6% of the uncertainty about its state on a random map.

**DISCUSSION**

What information is provided by each of these methods? And what is the relative cost to calculate each one? A brief summary is presented in Table 7. The indices of spatial pattern (fractal dimension, nearest neighbors probabilities, contagion, edges, spatial predictability) give statistical measures of whether certain aspects of the pattern are comparable, but do not indicate whether the cell by cell pattern is exactly (or approximately) matched. This may be adequate when one wishes to predict overall behavior of a system, or

<table>
<thead>
<tr>
<th>Measure</th>
<th>Characteristic</th>
<th>Computation</th>
<th>Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fractal dimension</td>
<td>complexity of spatial pattern</td>
<td>identify size and area of each path (time and memory intensive; can be unreliable for small patches)</td>
<td>broad-scale measure of pattern; may reflect scale of processes creating pattern</td>
</tr>
<tr>
<td>Nearest neighbor probabilities</td>
<td>adjacency and anisotropy</td>
<td>single pass through the matrix (fast, even for large matrices)</td>
<td>fine-scale measure of pattern and directionality</td>
</tr>
<tr>
<td>Contagion index</td>
<td>fragmentation or clumping</td>
<td>uses the probabilities of adjacency (fast)</td>
<td>overall measure; can also identify anisotropy</td>
</tr>
<tr>
<td>Edges</td>
<td>edge between components</td>
<td>single pass through the matrix (fast)</td>
<td>when exact amount of interface is important</td>
</tr>
<tr>
<td>Spatial predictability</td>
<td>departure from random; periodicity at different scales</td>
<td>uses n-th order probabilities of adjacency</td>
<td>departure from random on a (0, 1) scale; scale-dependent patterns</td>
</tr>
<tr>
<td>Multiple resolution fitting</td>
<td>correspondence to actual locations</td>
<td>calculates goodness-of-fit with decreasing resolution</td>
<td>pattern matching when accurate spatial locations are required between matrices</td>
</tr>
</tbody>
</table>
when these pattern aspects are otherwise important. The fractal dimension measures pattern complexity, providing information about patch shape, but does not address the adjacency of different cover types. The fractal has also been hypothesized to reflect the scale of the factors causing the pattern (Krummel et al., 1987). Nearest neighbor probabilities provide a fine-scale measure of adjacency patterns and the directionality of individual cover types. Nearest neighbor probabilities also reflect the degree of fragmentation in the landscape and, indirectly, the complexity of patch boundaries. The contagion index measures the dissection or clumping in the spatial pattern, but does not identify the factors causing the pattern. Edge calculations provide the amount of interface between different categories, but they are not sensitive to specific spatial arrangements. Spatial predictability measures the degree of departure from a random pattern and identifies periodicities in spatial patterns at different scales. First-order spatial adjacency predictability and the contagion index are highly correlated ($r^2 = 0.99$), but predictability may be easier to interpret because it is scaled on the range $[0, 1]$.

The multiple resolution fitting method compares spatial data on a cell by cell basis, but it is not sensitive to qualitative differences between spatial patterns. It may be particularly useful when the patterns are only slightly different, and if the patterns are properly registered. An advantage of the multiple resolution measure is its sensitivity to a periodic patterns or clusters and its lack of sensitivity to pattern complexity. The method can be used if simulating the actual location of categorical data is important. However, because of its insensitivity to qualitative aspects of the pattern (e.g., clumped versus dissected, complex versus simple), another indicator of spatial pattern might be used to supplement multiple resolution fitting if these aspects of the pattern are important.

Additional characteristics must be considered when selecting a method for comparing simulated and actual spatial patterns. The matrices should be of the same dimensions, because matrix size has been shown to affect the number, size, and shape (as measured by the fractal dimension) of clusters (Gardner et al., 1987). The grid cells should also be of the same resolution, or grain size, as pattern measures have been shown to vary with scale (Turner et al., in press). Data sets should contain the same number of data categories if indices based on information theory (e.g., contagion, spatial predictability) are to be used. Matrices must also be properly registered to obtain valid results from the multiple resolution fitting method. Computational considerations may also be important. For example, calculating fractal dimensions can require considerable computer memory and time with large matrices, because each patch in a matrix must be located and its area and perimeter recorded. In contrast, adjacency, contagion, edges and predictability can all be calculated on a single pass through the matrix.

The issue of significance differences in spatial patterns is critically significant differences in spatial patterns, and S. Turner for the available methods. However, patterns may result in subtle aspects are not well known (Gardner et al., 1987; Turner, 1987) governing whether or not system behavior. It is a concern, but significance of differences in patterns, predictability may be easier to interpret because it is scaled on the range $[0, 1]$.

The methods we have presented will depend on spatial patterns. Additional research can improve the evaluation of pattern predictability to simulate broad-scale and ultimately manage nature.

ACKNOWLEDGEMENTS

We thank Robert H. Gardner for comments on the manuscript. This work was supported by the Ecological Research Division, U.S. Department of the Interior, NOS, 84OR21400 with Martin M. H. Tolman and F.H. Sklar was project no. 14-16-0009-84-921 between the Coastal Ecology Institute of Hofstra University. This is Publication No. 8 of the Island National Laboratory.

REFERENCES

The fractal dimension about patch shape, but types. The fractal has also factors causing the pattern shibilities provide a fine-scale reality of individual cover degree of fragmentation of patch boundaries. The ring in the spatial pattern, pattern. Edge calculations in the pattern, but they are al predictability measures . Identifies periodicities in spatial adjacency predictability \( r^2 = 0.99 \), but predictabilit- on the range \( (0, 1) \).

Spatial data on a cell by differences between spatial patterns are only slightly red. An advantage of the periodic patterns or clusters the method can be used if \( \alpha \) is important. However, the pattern (e.g., clumped indicator of spatial pattern fitting if these aspects of when selecting a method is. The matrices should be been shown to affect the (dimension) of clusters be of the same resolution, hown to vary with scale the same number of data (e.g., contagion, spatial be properly registered to fitting method. Computer For example, calculating error memory and time with ist be located and its area contagion, edges and pre- through the matrix.

The issue of significant (both statistically and ecologically) changes or differences in spatial patterns remains an important research topic. Statistically significant differences in spatial data can be determined by a variety of techniques, and S. Turner et al. (in press) have recently reviewed many of the available methods. However, slight differences or changes in spatial patterns may result in substantial alterations in ecological processes, but this aspect is not well known. The existence of thresholds in spatial patterns (Gardner et al., 1987; Turner et al., 1989) may be quite important in governing whether or not a change in pattern results in qualitatively different system behavior. It also remains necessary to determine the ecological significance of differences in broad-scale pattern indices (e.g., contagion, fractal dimension, predictability). Additional research is required to elucidate these topics.

The methods we have presented are useful for examining the goodness-of-fit between spatial simulations and data. The selection of appropriate methods will depend on specific modeling objectives and on the attributes of the data. Additional research in pattern analysis and comparison should improve the evaluation of landscape models, and therefore enhance our ability to simulate broad-scale phenomena, characterize spatial patterns, and ultimately manage natural resources at the landscape level.

ACKNOWLEDGEMENTS

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REFERENCES


Energy Intensity, Power, and Ascendency in Ecosystems

ROBERT HERENDDEEN
Illinois Natural History Survey, Urbana, Illinois
(Received 10 April 1989)

ABSTRACT


Several system-wide indicators have been proposed as optimands in the concomitant compartment stocks vary over time. The proposed indicators vary over time and indicators will show different scales of changes of availability of energy. The one(s) deserve further study. Apparent real steady-state data indicate that ascendency least sensitive. Exergy problems of system boundary, can be therefore discussed at some length presented.

1. INTRODUCTION

Several quantities (energy, exergy, power) proposed for multipurpose, on a system level, the distribution of natural flows between them. Type classification for steady-state ecosystem. Patten and Finn, 1979; Urban, 1985.

All three of the quantities are principles which can be used in perturbed ecosystems (review references below). Even though

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